Polynomials

Quick Revision

A **polynomial** in one variable x, is an algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

where n is a positive integer and constants $a_0, a_1, a_2, \dots, a_n$ are known as **coefficients of polynomial**.

Degree of a Polynomial

The highest power (exponent) of x in a polynomial $f(x)$, is called the degree of the polynomial $f(x)$.

e.g. $g(y) = 3y^2 - \frac{5}{2}y + 7$ is a polynomial in variable y of degree 2.

Types of Polynomials

(i) **Linear Polynomial** A polynomial of degree one, is called linear polynomial.

e.g. $f(x) = 3x + 5$

(ii) **Quadratic Polynomial** A polynomial of degree two, is called quadratic polynomial.

e.g. $f(x) = 5x^2 + 3x - \frac{7}{5}$

(iii) **Cubic Polynomial** A polynomial of degree three, is called cubic polynomial.

e.g. $f(x) = 9x^3 + 5x^2$

(iv) **Biquadratic Polynomial** A polynomial of degree four, is called biquadratic polynomial.

e.g. $f(x) = x^4 + 2x^3 - 7x^2 + 5x + 3$

Value of a Polynomial at Given Point

If $p(x)$ is a polynomial and α is a real value, then the value obtained by putting $x = \alpha$ in $p(x)$, is called the value of $p(x)$ at $x = \alpha$ and it is denoted by $p(\alpha)$.

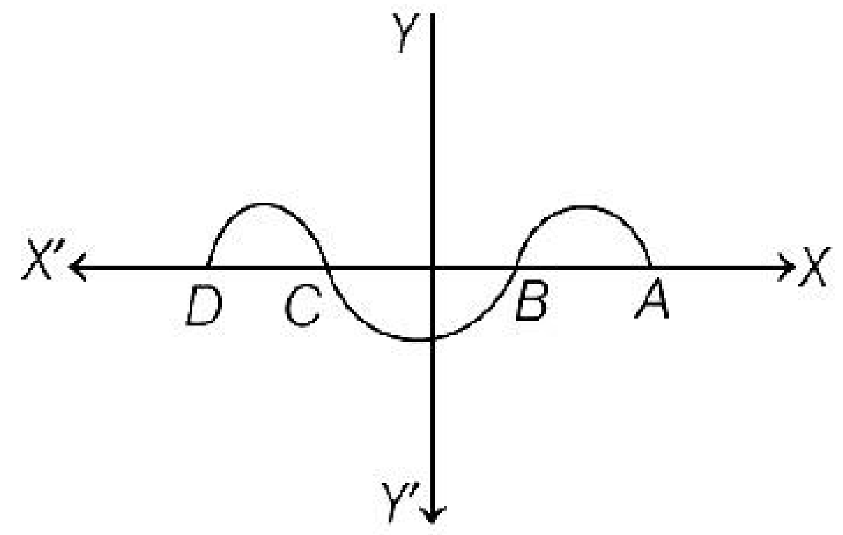
Zeroes of a Polynomial

A real number k is said to be a zero of a polynomial $f(x)$, if $f(k) = 0$.

Geometrical Meaning of the Zeroes of a Polynomial

The geometrical meaning of the zeroes of a polynomial means that the curve intersect the X-axis, the intersection point is said to be zeroes of the curve.

e.g. In the figure we see that graph intersect the X-axis at points A, B, C and D. So, it has four zeroes.



Relationship between Zeroes and Coefficients of a Polynomial

The zeroes of a polynomial are related to its coefficients.

(i) **For a Linear Polynomial** The zero of the linear polynomial $ax + b$ is

$$-\frac{b}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x}.$$

(ii) **For a Quadratic Polynomial** Let α and β be the zeroes of quadratic polynomial $p(x) = ax^2 + bx + c, a \neq 0$, then

$\therefore$ Sum of zeroes, $\alpha + \beta$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$$

and product of zeroes, $\alpha\beta$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$$

(iii) **For a Cubic Polynomial** Let α, β and γ be the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d, a \neq 0$, then

$$\alpha + \beta + \gamma = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$$

and $$\alpha\beta\gamma = -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{d}{a}$$

Useful Algebraic Identities

(i) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(ii) $(a + b)^2 - (a - b)^2 = 4ab$ and $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(iii) $(a^2 - b^2) = (a + b)(a - b)$

(iv) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$

(v) $(a^3 \pm b^3) = (a \pm b)(a^2 + b^2 \mp ab)$
$= (a \pm b)[(a \pm b)^2 \mp 3ab]$

(vi) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
$= a^3 + b^3 + 3a^2b + 3ab^2$

(vii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
$= a^3 - b^3 - 3a^2b + 3ab^2$

(viii) $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Formation of Quadratic and Cubic Polynomials

(i) If α and β are the zeroes of a quadratic polynomial, then quadratic polynomial will be
$k\,[x^2 -$ (sum of zeroes) $x +$ product of zeroes$]$

i.e. $k\,[x^2 - (\alpha + \beta)x + \alpha\beta]$, where k is some constant.

(ii) If α, β and γ are the zeroes of cubic polynomial, then cubic polynomial will be
$k[x^3 -$ (sum of zeroes) x^2
$+$ (sum of the product of zeroes taking two at a time)x
$-$ product of zeroes$]$

i.e. $k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$

Objective Questions

Multiple Choice Questions

1. Which of the following is not a polynomial

(a) $3x+5$ (b) $3y^3-4y^2+2y$

(c) x^3-3 (d) $\frac{1}{x+2}$

2. If 2 is a zero of polynomial $f(x)=ax^2-3(a-1)x-1$, then the value of a is

(a) 0 (b) 2

(c) $\frac{5}{2}$ (d) $\frac{1}{2}$

3. If one of the zeroes of the quadratic polynomial $(k-1)x^2+kx+1$ is -3, then the value of k is *[NCERT Exemplar]*

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$

(c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

4. If one of the zeroes of the quadratic polynomial x^2+3x+k is 2, then the value of k is *[CBSE 2020]*

(a) 10 (b) -10

(c) -7 (d) -2

5. If 2 and 3 are zeroes of polynomial $3x^2-2kx+2m$, then the values of k and m are

(a) $m=\frac{9}{2}$ and $k=15$ (b) $m=\frac{15}{2}$ and $k=9$

(c) $m=9$ and $k=\frac{15}{2}$ (d) $m=15$ and $k=9$

6. The value of p, for which (-4) is a zero of the polynomial $x^2-2x-(7p+3)$ is ....

(a) 3 (b) 2

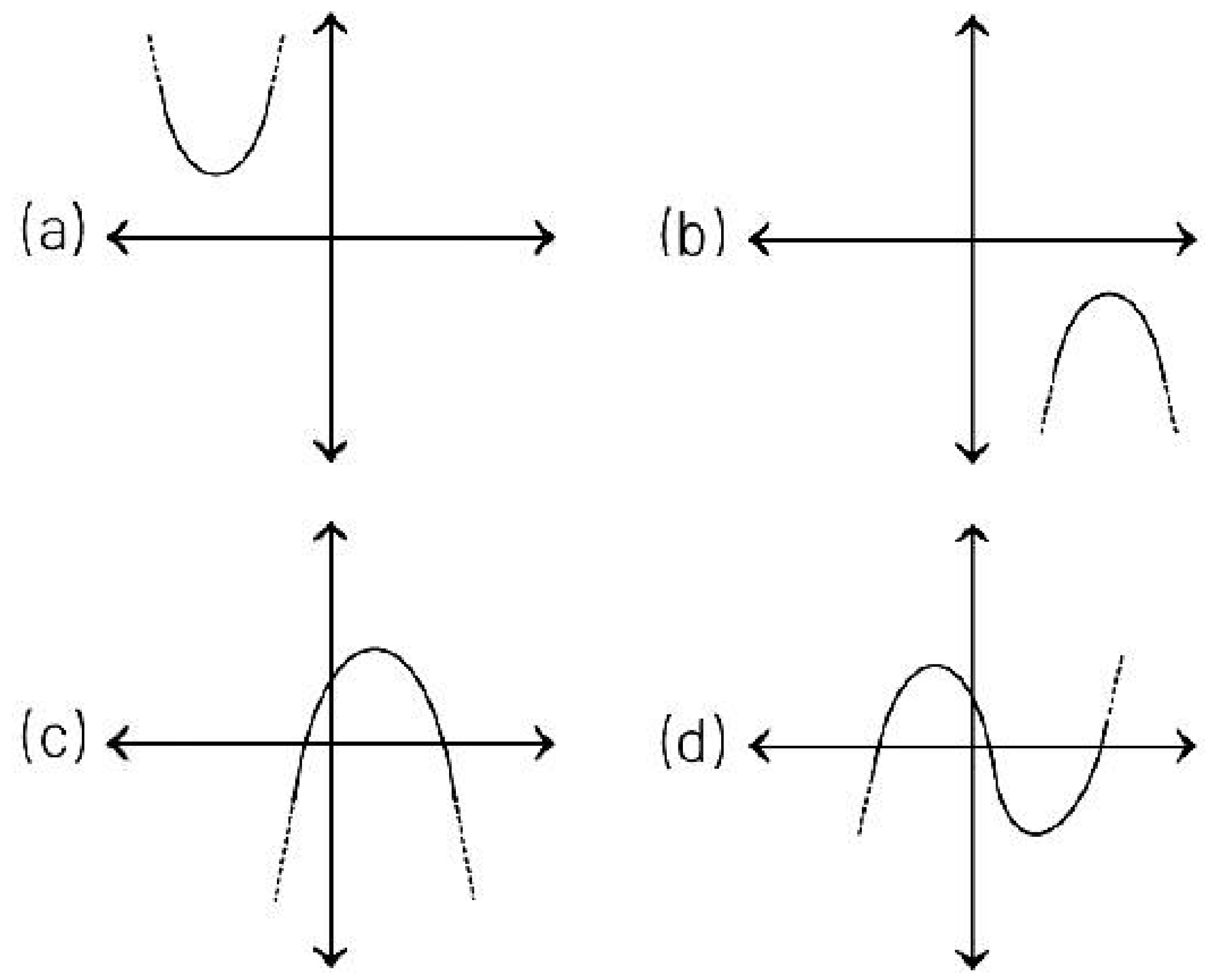
(c) 4 (d) -2

7. If the graph of a polynomial intersects the X-axis at only one point, it can be a

(a) linear (b) quadratic

(c) cubic (d) None of these

8. Which of the following is not the graph of a quadratic polynomial?



9. The graph of a quadratic polynomial is .... *[NCERT Exemplar]*

(a) straight line (b) parabola

(c) hyperbola (d) None of these

10. If one zero of polynomial x^2-4x+1 is $2+\sqrt{3}$, then other zero will be ....

(a) $-2+\sqrt{3}$ (b) $-\sqrt{3}-2$

(c) $2-\sqrt{3}$ (d) $\sqrt{3}+1$

11. The zeroes of the quadratic polynomial (x^2+5x+6) are

(a) -2 and -3 (b) 3 and 4

(c) 3 and 2 (d) 2 and 1

12. Zeroes of $p(z)=z^2-27$ are .... and ....

(a) $\pm 3\sqrt{3}$ (b) $+3$

(c) $+9$ (d) $+\sqrt{3}$ and $-\sqrt{3}$

13. The zeroes of the quadratic polynomial $f(x)=abx^2+(b^2-ac)x-bc$ are

(a) $\frac{b}{ac}$ and $\frac{c}{b}$ (b) $\frac{ab}{c}$ and $\frac{a}{b}$

(c) $\frac{-b}{a}$ and $\frac{c}{b}$ (d) $\frac{b}{a}$ and $\frac{-c}{b}$

14. The number of polynomials having zeroes as -2 and 5 is *[NCERT Exemplar]*

(a) 1 (b) 2
(c) 3 (d) more than 3

15. 1 and 2 are the zeroes the polynomial $x^2 - 3x + 2$

(a) True (b) False
(c) Can't say (d) Partially true/false

16. Every real number is the zeroes of zero polynomial

(a) True (b) False
(c) Can't say (d) Partially true/False

17. $p(x) = x - 1$ and $g(x) = x^2 - 2x + 1$, $p(x)$ is a factor of $g(x)$

(a) True (b) False
(c) Can't Say (d) Partially True/False

18. The value of k for which 3 is a zero of polynomial $2x^2 + x + k$ is ....

(a) 21 (b) 20
(c) -21 (d) 18

19. If zeroes α and β of a polynomial $x^2 - 7x + k$ are such that $\alpha - \beta = 1$, then the value of k is

(a) 21 (b) 12
(c) 9 (d) 8

20. Sum of zeroes of Quadratic polynomial

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(a) True (b) False
(c) Can't say (d) Partially true/false

21. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$ the value of 'a', if $3\alpha + 2\beta = 20$ is -12

(a) True (b) False
(c) Can't say (d) Partially True/False

22. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative
(b) has no linear term and the constant term is positive
(c) can have a linear term but the constant term is negative
(d) can have a linear term but the constant term is positive

23. If p and q are zeroes of $3x^2 + 2x - 9$, then value of $p - q$ is

(a) -3 (b) $-\frac{2}{3}$
(c) $\pm\frac{4\sqrt{7}}{3}$ (d) None of these

24. The polynomial whose zeroes are $(\sqrt{2}+1)$ and $(\sqrt{2}-1)$ is

(a) $x^2 + 2\sqrt{2}x + 1$ (b) $x^2 - 2\sqrt{2}x + 1$
(c) $x^2 + 2\sqrt{2}x - 1$ (d) $x^2 - 2\sqrt{2}x - 1$

25. If α and β are the zeroes of the polynomial $2y^2 + 7y + 5$, then the value of $\alpha + \beta + \alpha\beta$ is

(a) -1 (b) 0
(c) 1 (d) 2

26. If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 5x - 2$, then $\alpha^3 + \beta^3$ is equal to

(a) $\frac{215}{27}$ (b) $\frac{357}{21}$
(c) $\frac{115}{28}$ (d) $\frac{325}{31}$

27. If α and β are the zeroes of $4x^2 + 3x + 7$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

(a) $-\frac{8}{7}$ (b) $-\frac{3}{7}$
(c) $\frac{2}{7}$ (d) $\frac{6}{8}$

28. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then *[NCERT Exemplar]*

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$
(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

29. If the sum and difference of zeroes of quadratic polynomial are – 3 and – 10, respectively. Then, the difference of the squares of zeroes is

(a) 20 (b) 30
(c) 15 (d) 25

30. If sum and product of zeroes of quadratic polynomial are, respectively 8 and 12, then their zeroes are

(a) 2 and 6 (b) 3 and 4
(c) 2 and 8 (d) 2 and 5

31. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, then find the value of $\frac{m}{n} + \frac{n}{m}$

(a) $\frac{145}{12}$ (b) $-\frac{145}{12}$
(c) $\frac{145}{7}$ (d) $\frac{-145}{15}$

32. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, then the value of k is

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$
(c) $\frac{2}{7}$ (d) $\frac{3}{2}$

33. If sum of the squares of zeroes of the quadratic polynomial $f(x) = x^2 - 4x + k$ is 20, then the value of k is

(a) –2 (b) –3
(c) –4 (d) 2

34. If α and β are zeroes of the polynomial $x^2 - p(x+1) + c$ such that $(\alpha+1)(\beta+1) = 0$, then the value of c is

(a) –2 (b) 2
(c) –1 (d) 1

35. The value of k such that the polynomial $x^2 - (k+6)x + 2(2k-1)$ has sum of its zeroes equal to half of their product is **[CBSE 2019]**

(a) –4 (b) 4 (c) –7 (d) 7

36. The sum and the product of zeroes of the polynomial $f(x) = 4x^2 - 27x + 3k^2$ are equal the value of k is

(a) $k = 3$ (b) $k = -3$
(c) $k = \pm 3$ (d) $k = 2$

37. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4x + 3$, then the value of $\alpha^4\beta^3 + \alpha^3\beta^4$ is

(a) 104 (b) 108
(c) 112 (d) 5

38. The quadratic polynomial, the sum of whose zeroes is – 5 and their product is 6, is **[CBSE 2020]**

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$
(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

39. The quadratic polynomial whose zeroes are $2\sqrt{7}$ and $-5\sqrt{7}$ is

(a) $x^2 - 3\sqrt{7}x - 70$ (b) $x^2 + 3\sqrt{7}x + 70$
(c) $x^2 + 3\sqrt{7}x - 70$ (d) $x^2 - 3\sqrt{7}x + 70$

40. The quadratic polynomial, whose zeroes are $3 + \sqrt{2}$ and $3 - \sqrt{2}$, is

(a) $x^2 - 3x + 5$ (b) $x^2 - 6x + 7$
(c) $x^2 - 7x + 6$ (d) $x^2 - 8x + 12$

41. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + x - 2$, then the polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$ is

(a) $x^2 + 9$ (b) $x^2 - 4$
(c) $x^2 - 9$ (d) $x^2 + 4$

42. If α and β are zeroes of a quadratic polynomial $x^2 - 5$, then the quadratic polynomial whose zeroes are $1+\alpha$ and $1+\beta$ is

(a) $x^2+2x+24$ (b) $x^2-2x-24$
(c) $x^2-2x+24$ (d) None of these

43. The number of value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is

(a) 4 (b) 1 (c) 2 (d) 3

44. $p(x) = 5x^3 - 3x^2 + 7x + 2$, then match the value of Column I with that of Column II

	Column I		Column II
A.	$p(1)$	P.	2
B.	$p(0)$	Q.	11
C.	$p(5)$	R.	-13
D.	$p(-1)$	S.	-64
E.	$p(-2)$	T.	587

	A	B	C	D	E		A	B	C	D	E
(a)	Q	T	R	P	S	(b)	Q	R	T	S	P
(c)	Q	P	T	R	S	(d)	T	R	Q	S	P

45. If α and β are the zeroes of the polynomial $2x^2 - 4x + 5$, then match the value of Column I with that of Column II

	Column I		Column I
A.	$\frac{1}{\alpha} + \frac{1}{\beta}$	1.	-6
B.	$(\alpha - \beta)^2$	2.	$\frac{-4}{25}$
C.	$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$	3.	$\frac{-2}{5}$
D.	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$	4.	$\frac{4}{5}$

	A	B	C	D		A	B	C	D
(a)	4	1	2	3	(b)	4	2	1	3
(c)	1	2	3	4	(d)	1	4	2	3

Assertion-Reasoning MCQs

Directions (Q. Nos. 46-55) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is False.
(d) A is false; R is true.

46. **Assertion (A)** $x^2 + 4x + 5$ has two zeroes.

Reason (R) A quadratic polynomial can have atmost two zeroes.

47. **Assertion (A)** The factor of $\frac{3}{2}x^2 - 8x - \frac{35}{2}$ is $\frac{1}{2}(x - 7)(3x + 5)$

Reason (R) The factors are calculated by dividing the coefficients by 2 and expression is obtained by splitting the middle term

48. **Assertion (A)** A quadratic polynomial having 4 and 3 as zeroes is $x^2 - 7x - 12$

Reason (R) The quadratic polynomial having α and β as zeroes is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha \cdot \beta$

49. **Assertion (A)** Zeroes of $f(x) = x^2 - 4x - 5$ are $5, -1$.

Reason (R) The polynomial whose zeroes are $2 + \sqrt{3}, 2 - \sqrt{3}$ is $x^2 - 4x + 7$

50. **Assertion (A)** If α and β are the zeroes of the polynomial $x^2 + 2x - 15$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is $\frac{2}{15}$.

Reason (R) If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

51. **Assertion (A)** If one zero of the polynomial $p(x) = (k + 18)x^2 + 11x + 4k$ is the reciprocal of the other zero, then $k = 6$

Reason (R) If $(x-\alpha)$ and $(x-\beta)$ are the factor of the polynomial $p(x)$, then α and β are the zeroes of the $p(x)$

52. **Assertion (A)** If the zeroes of x^2+px+q are two consecutive integers, then $p^2-1=4q$

Reason (R) If α, β are zeroes of $(x-a)(x-b)-c$, then a, b are zeroes of $(x-\alpha)(x-\beta)+c$

53. **Assertion (A)** If the zeroes of the polynomial $f(x)=(k^2+4)x^2+4kx+(k^3-9)$ are equal in magnitude but opposite in sign, then value of k is zero

Reason (R) A quadratic polynomial whose zeroes are α and β is $x^2+(\alpha+\beta)x+\alpha\beta$

54. **Assertion (A)** If m and n are the zeroes of the polynomial $3x^2+11x-4$, then $12(m^2+n^2)+145mn=0$

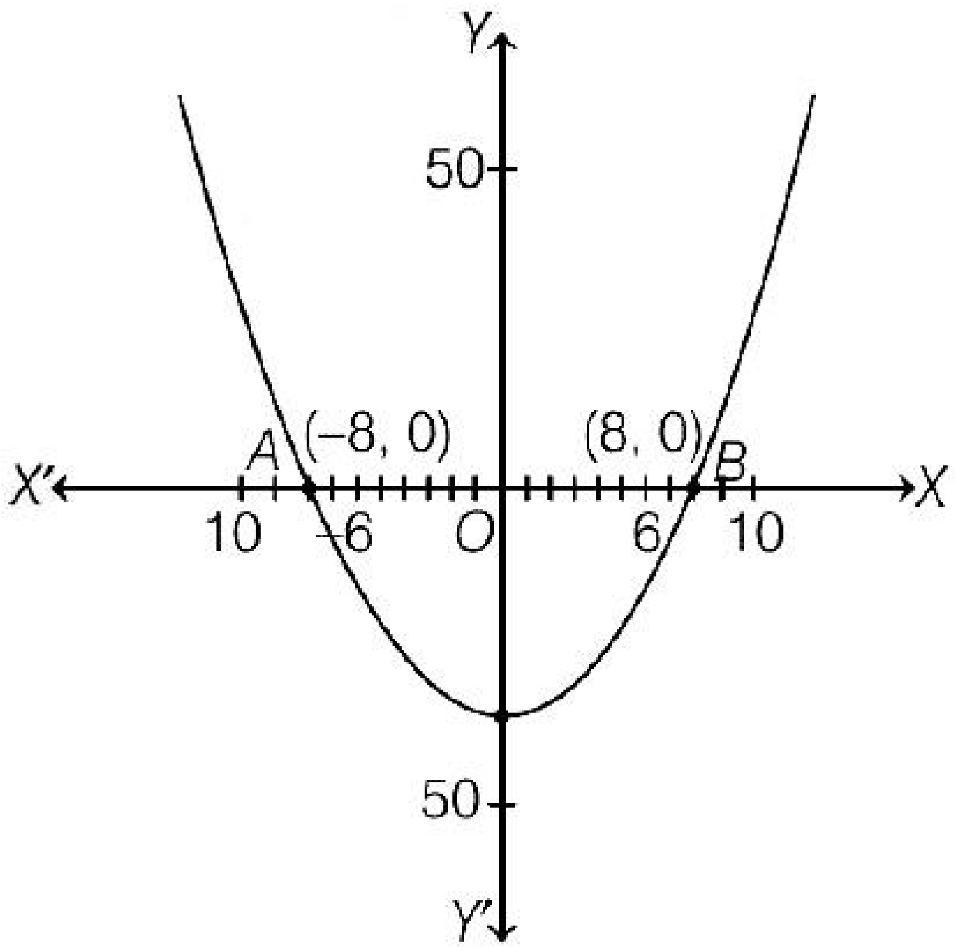
Reason (R) If α and β are the zeroes of the polynomial ax^2+bx+c, then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{b^2}{ac}+2$

55. **Assertion (A)** If x^2+x-12 divides $p(x)=x^3+ax^2+bx-84$ exactly, then $a=-8$ and $b=-5$

Reason (R) When a polynomial $p(x)$ is completely divided by $(x-\alpha)$, then $p(x)=0$.

Case Based MCQs

56. While playing in garden, Rohit saw a Nest and asked her elder brother what is that he replied that it's a nest made by bird to live themselves. Also he told him that the shape of the nest formed is parabolic. The mathematical representation of the nest structure is shown in the graph.



Based on the above information, answer the following questions.

(i) Graph of a quadratic polynomial is ______ in shape.
(a) straight line (b) parabolic
(c) circular (d) None of these

(ii) The expression of the polynomial represented by the graph is
(a) x^2-49
(b) x^2-64
(c) x^2-36
(d) x^2-81

(iii) Find the value of the polynomial represented by the graph when $x=8$
(a) -2 (b) -1
(c) 0 (d) 1

(iv) The sum of zeroes of the polynomial x^2-2x-4 is
(a) -1 (b) -2
(c) 2 (d) 1

(v) If the sum of zeroes of polynomial $ax^2 + 5x - 3a$ is equal to their product, then find the value of a

(a) -5 (b) -3

(c) $-\frac{5}{3}$ (d) $\frac{5}{3}$

57. The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola.

In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms. ***[CBSE Question Bank]***

Based on the above information, answer the following questions.

(i) In the standard form of quadratic polynomial, $ax^2 + bx + c$, a, b and c are

(a) All are real numbers

(b) All are rational numbers.

(c) 'a' is a non-zero real number, b and c are any real numbers.

(d) All are integers.

(ii) If the roots of the quadratic polynomial are equal, where the discriminant $D = b^2 - 4ac$, then

(a) $D > 0$

(b) $D < 0$

(c) $D \geq 0$

(d) $D = 0$

(iii) If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then k is

(a) 4 (b) $\frac{1}{4}$

(c) $\frac{-1}{4}$ (d) 2

(iv) The graph of $x^2 + 1 = 0$

(a) Intersects X-axis at two distinct points.

(b) Touches X-axis at a point.

(c) Neither touches nor intersects X-axis.

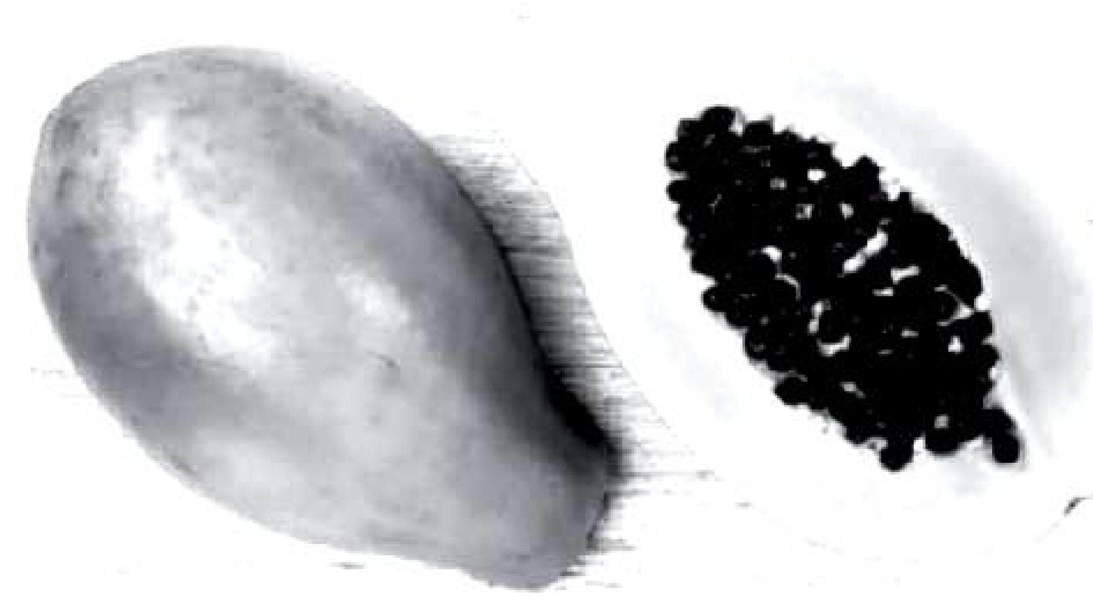
(d) Either touches or intersects X-axis.

(v) If the sum of the roots is $-p$ and product of the roots is $-\frac{1}{p}$, then the quadratic polynomial is

(a) $k\left(-px^2 + \frac{x}{p} + 1\right)$ (b) $k\left(px^2 - \frac{x}{p} - 1\right)$

(c) $k\left(x^2 + px - \frac{1}{p}\right)$ (d) $k\left(x^2 - px + \frac{1}{p}\right)$

58. Nirahua's father gave him some money to buy papaya from the market at the rate of $p(x) = x^2 - 23x + 120$. Let α, β are the zeroes of $p(x)$.



Based on the above information, answer the following questions.

(i) Find the value of α and β, where $\alpha < \beta$

(a) $-8, -16$ (b) 8, 16

(c) 8, 15 (d) 4, 9

(ii) Find the value of $\alpha + \beta + \alpha\beta$

(a) 151 (b) 158

(c) 143 (d) 155

(iii) The value of $p(2)$ is

(a) 80 (b) 81
(c) 83 (d) 78

(iv) If α and β are zeroes of $x^2 + 3x - 10$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

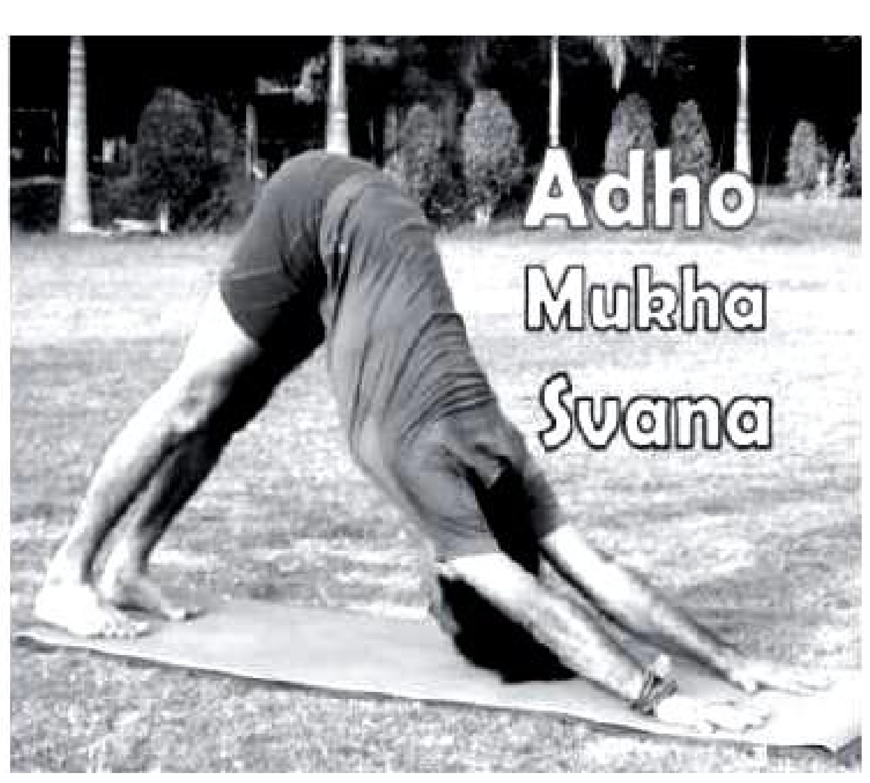
(a) 3/10 (b) 1/3
(c) 1/4 (d) 1/5

(v) If sum of zeroes of $q(x) = kx^2 + 2x + 3k$ is equal to their product, then $k =$

(a) 2/3 (b) 1/3
(c) $-2/3$ (d) $-1/3$

59. An asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial.

[CBSE Question Bank]



Based on the above information, answer the following questions.

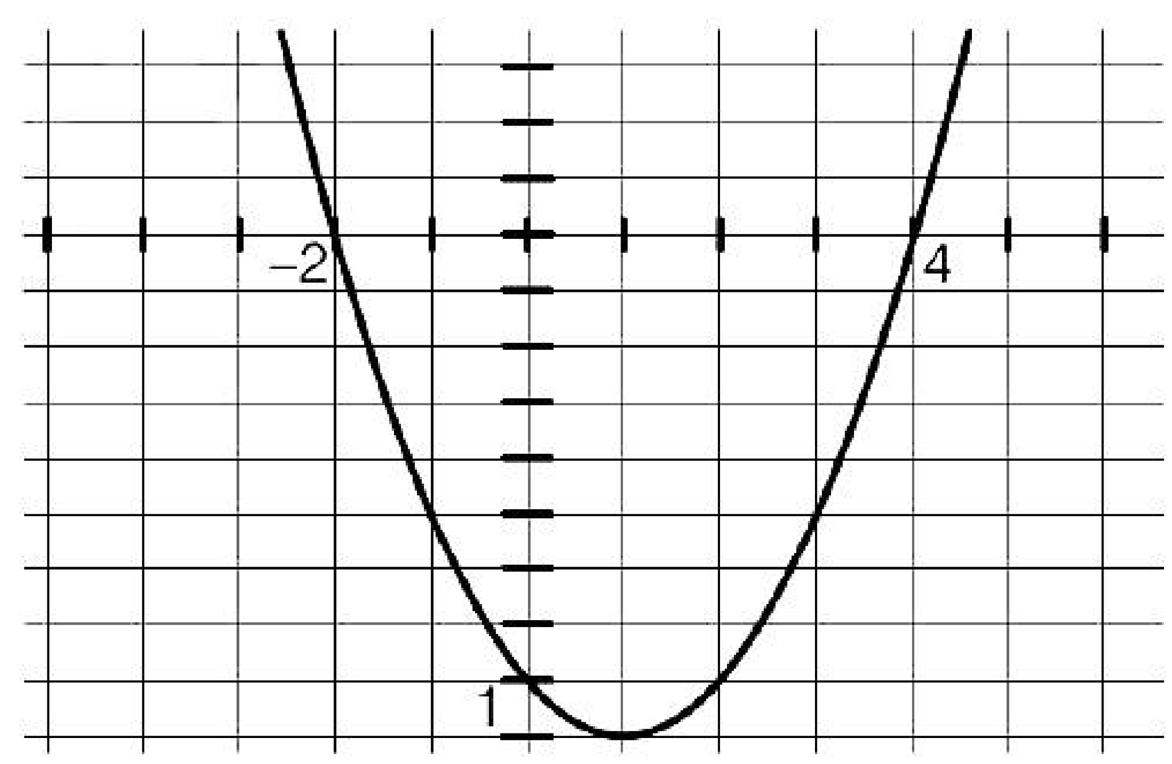
(i) The shape of the poses shown is

(a) Spiral
(b) Ellipse
(c) Linear
(d) Parabola

(ii) The graph of parabola opens downwards, if ....

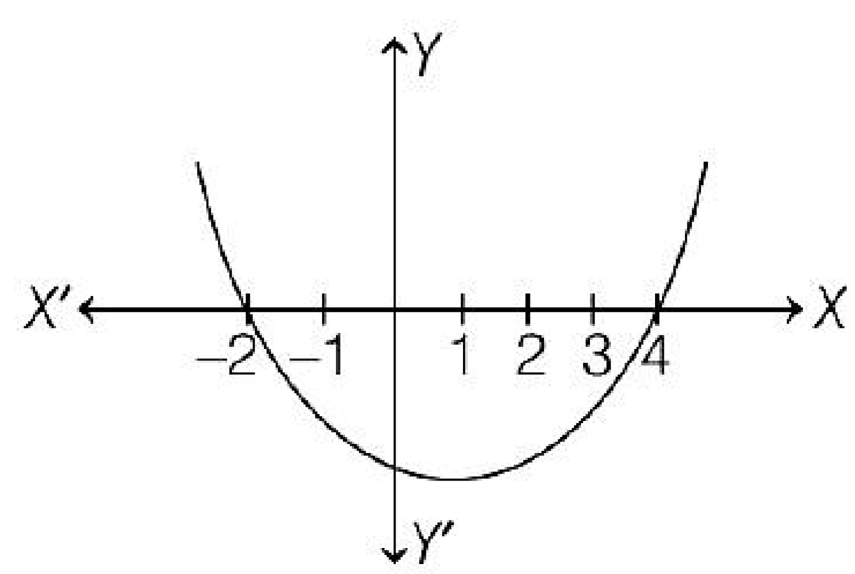
(a) $a \geq 0$ (b) $a = 0$
(c) $a < 0$ (d) $a > 0$

(iii) In the graph, how many zeroes are there for the polynomial?



(a) 0 (b) 1
(c) 2 (d) 3

(iv) The two zeroes in the below shown graph are

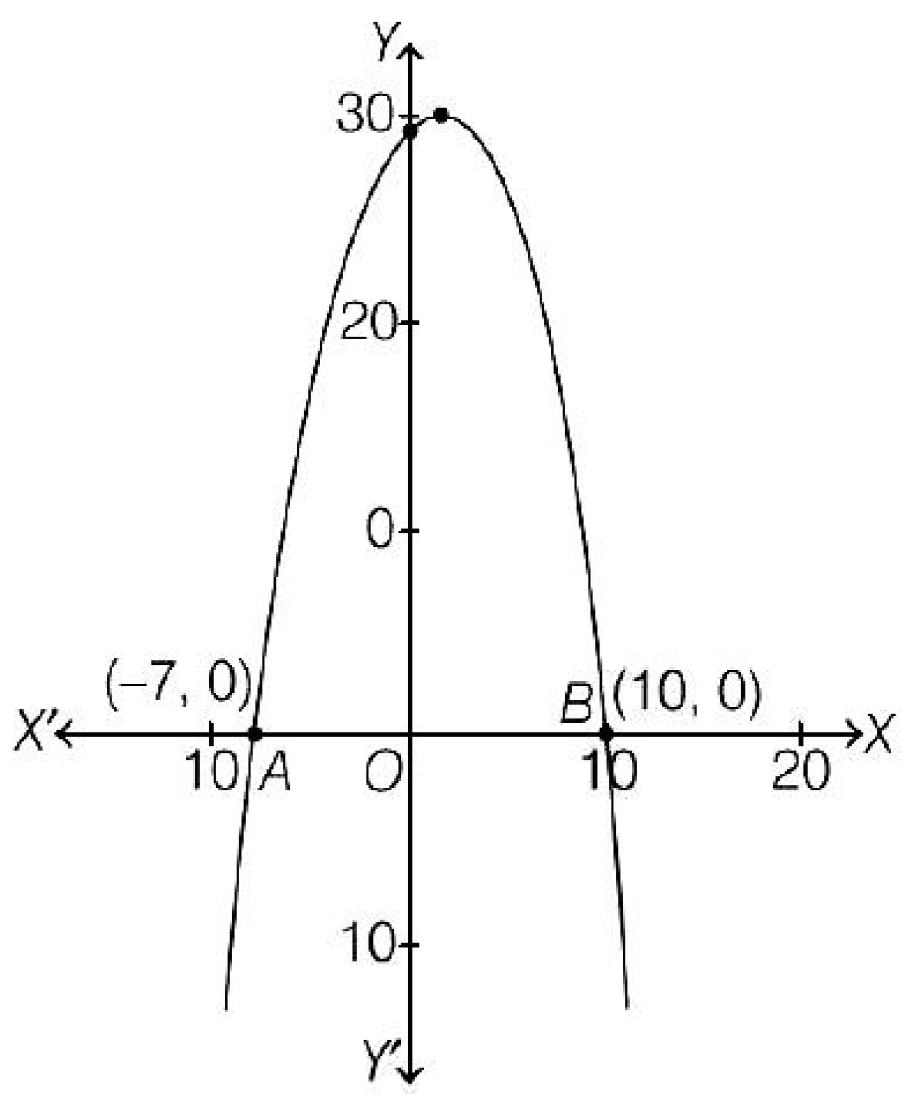


(a) 2, 4 (b) -2, 4
(c) -8, 4 (d) 2, -8

(v) The zeroes of the quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are

(a) $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (b) $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$
(c) $\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$ (d) $-\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

60. Two friend, Aanchal and Amarjeet during their summer vacations went to Rishikesh. They decided to go for trekking. While trekking they observes that the trekking path is in the shape of a parabola. The mathematical representation of the track is shown in the graph.



Based on the above information, answer the following questions.

(i) The zeroes of the polynomial whose graph is given are
(a) 4, 7 (b) -4, 7
(c) 4, 3 (d) -7, 10

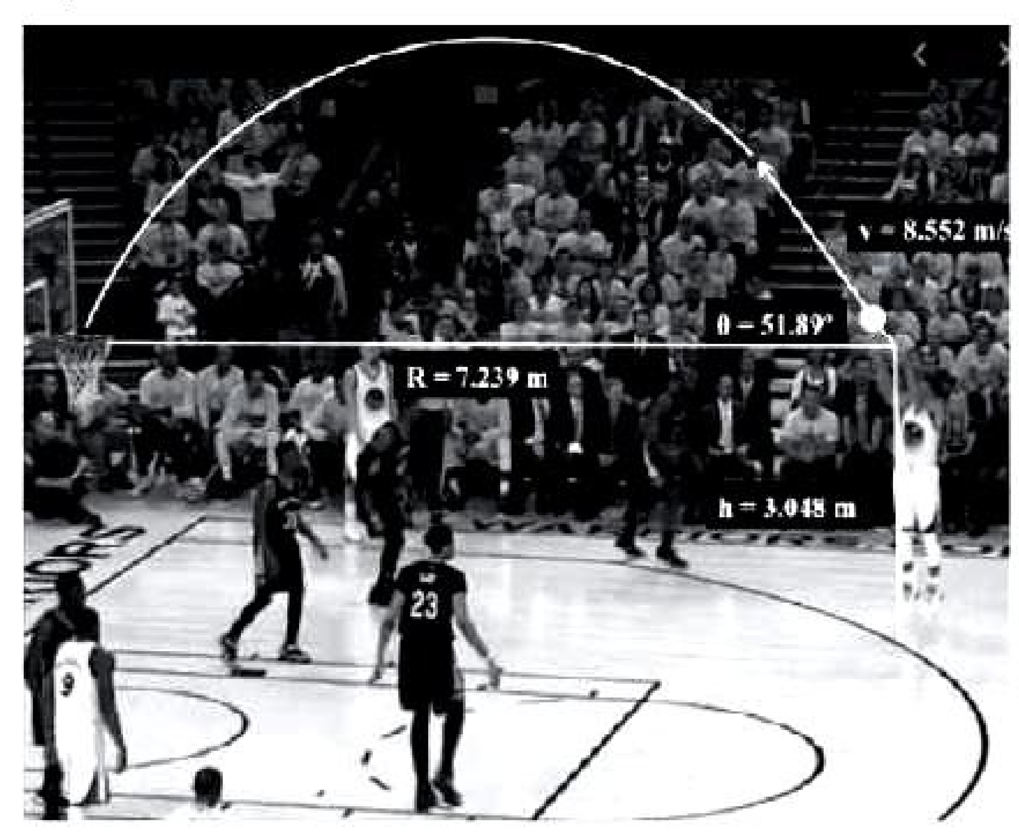
(ii) What will be the expression of the given polynomial $p(x)$
(a) $x^2 - 3x - 70$
(b) $-x^2 + 4x + 28$
(c) $x^2 - 4x + 28$
(d) $-x^2 + 3x + 28$

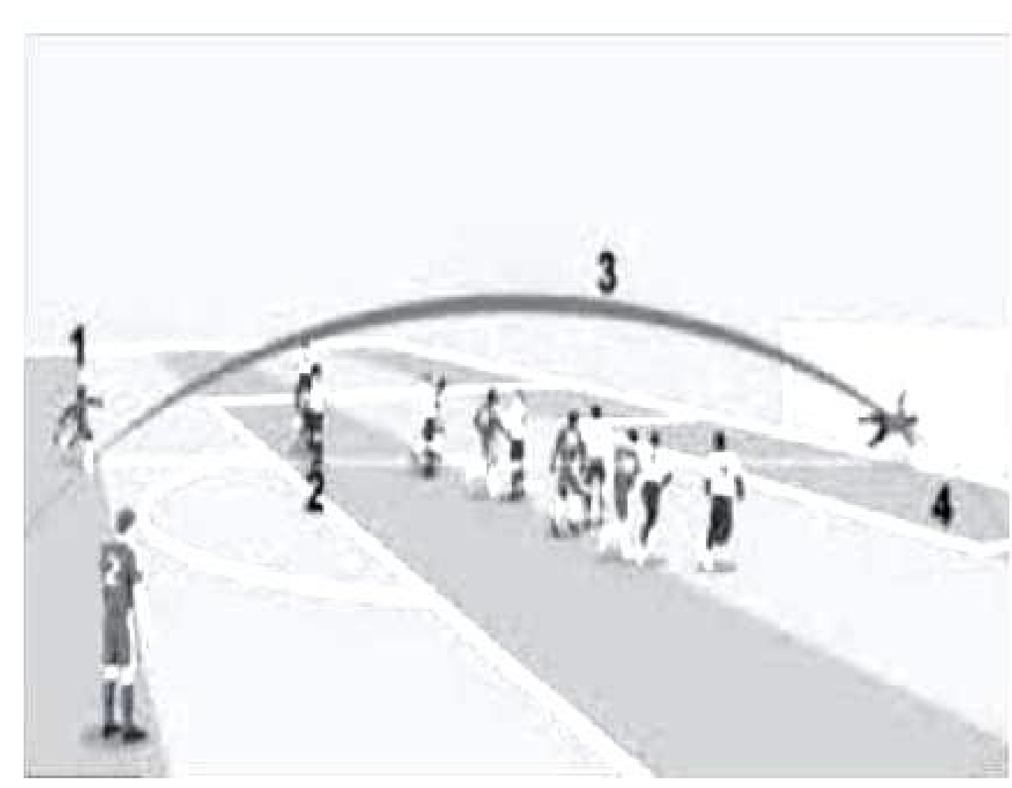
(iii) Product of zeroes of the given polynomial is
(a) -28 (b) 28
(c) -70 (d) 30

(iv) The zeroes of the polynomial $9x^2 - 5$ are
(a) $\frac{3}{\sqrt{5}}, \frac{-3}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$
(c) $\frac{\sqrt{5}}{3}, \frac{-\sqrt{5}}{3}$ (d) $\frac{\sqrt{5}}{2}, \frac{-\sqrt{5}}{2}$

(v) If $f(x) = x^2 - 13x + 1$, then $f(3)$ is
(a) 35 (b) -29
(c) 36 (d) -36

61. Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial. ***[CBSE Question Bank]***





Based on the above information, answer the following questions.

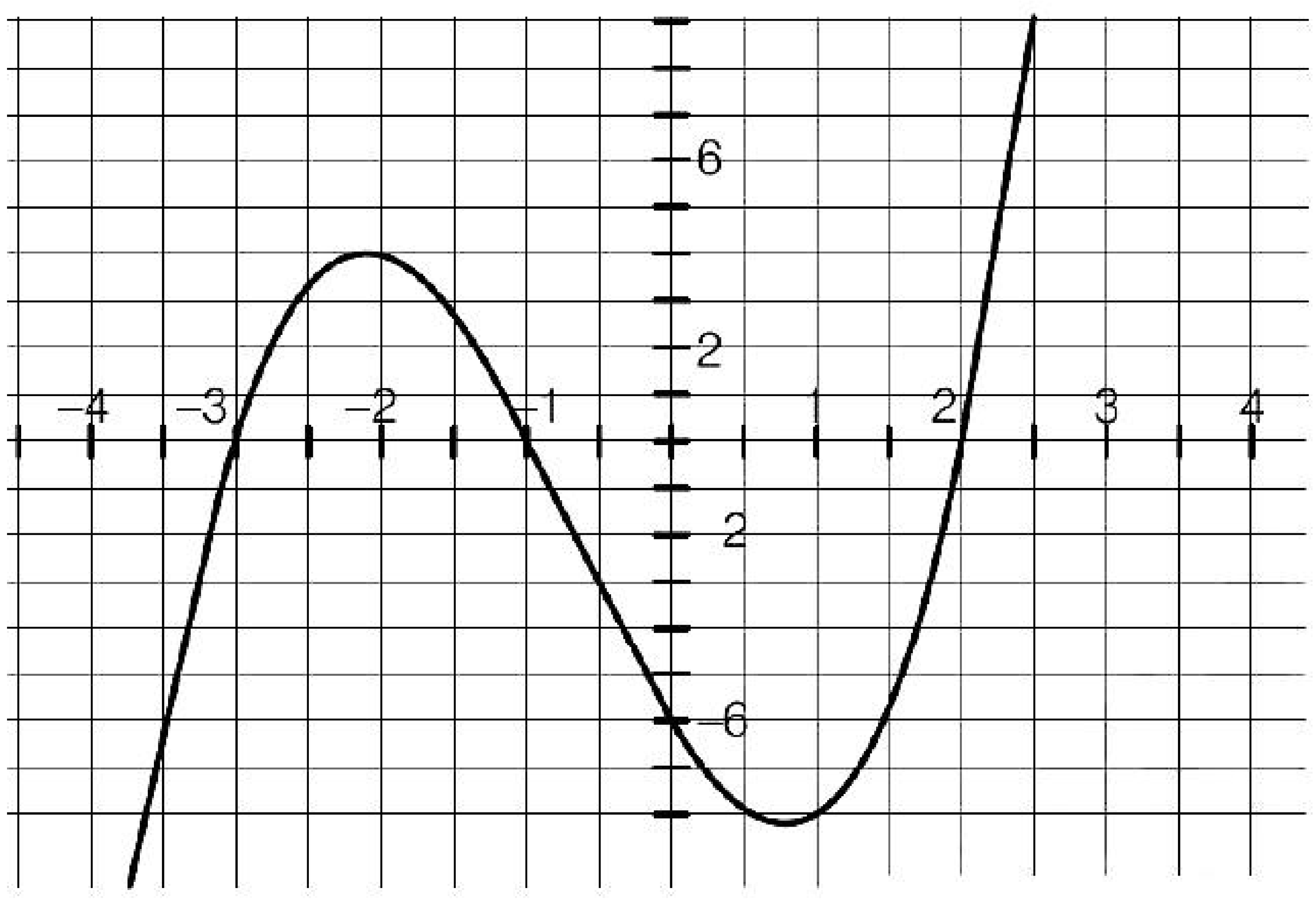
(i) The shape of the path traced shown is

(a) Spiral (b) Ellipse
(c) Linear (d) Parabola

(ii) The graph of parabola opens downwards, if ....

(a) $a = 0$ (b) $a < 0$
(c) $a > 0$ (d) $a \geq 0$

(iii) Observe the following graph and answer.



In the above graph, how many zeroes are there for the polynomial?

(a) 0
(b) 1
(c) 2
(d) 3

(iv) The three zeroes in the above shown graph are

(a) $2, 3, -1$
(b) $-2, 3, 1$
(c) $-3, -1, 2$
(d) $-2, -3, -1$

(v) What will be the expression of the polynomial?

(a) $x^3 + 2x^2 - 5x - 6$
(b) $x^3 + 2x^2 - 5x + 6$
(c) $x^3 + 2x^2 + 5x - 6$
(d) $x^3 + 2x^2 + 5x + 6$

ANSWERS

Multiple Choice Questions

1. (d)	2. (c)	3. (a)	4. (b)	5. (c)	6. (a)	7. (a)	8. (d)	9. (b)	10. (c)
11. (a)	12. (a)	13. (c)	14. (d)	15. (a)	16. (a)	17. (a)	18. (c)	19. (b)	20. (a)
21. (b)	22. (a)	23. (c)	24. (b)	25. (a)	26. (a)	27. (b)	28. (d)	29. (b)	30. (a)
31. (b)	32. (b)	33. (a)	34. (c)	35. (d)	36. (c)	37. (b)	38. (a)	39. (c)	40. (b)
41. (c)	42. (b)	43. (c)	44. (c)	45. (a)					

Assertion-Reasoning MCQs

46. (a)	47. (c)	48. (a)	49. (c)	50. (a)	51. (b)	52. (b)	53. (c)	54. (c)	55. (d)

Case Based MCQs

56. (i) - (b); (ii) - (b); (iii) - (c); (iv) - (c); (v) - (d)
57. (i) - (c); (ii) - (d); (iii) - (b); (iv) - (c); (v) - (c)
58. (i) - (c); (ii) - (c); (iii) - (d); (iv) - (a); (v) - (c)
59. (i) - (d); (ii) - (c); (iii) - (b); (iv) - (b); (v) - (b)
60. (i) - (d); (ii) - (a); (iii) - (c); (iv) - (c); (v) - (b)
61. (i) - (d); (ii) - (b); (iii) - (d); (iv) - (c); (v) - (a)

SOLUTIONS

1. $\frac{1}{x+2}$ is not a polynomial

Since, the exponent of variable is negative

2. Given, $f(x) = ax^2 - 3(a-1)x - 1$

and 2 is zero of $f(x)$,

$$f(2) = 0$$
$$f(2) = a(2)^2 - 3(a-1) \times 2 - 1$$
$$0 = 4a - 6a + 6 - 1$$
$$2a = 5$$
$$a = \frac{5}{2}$$

3. Given that, one of the zeroes of the quadratic polynomial say $p(x) = (k-1)x^2 + kx + 1$ is -3, then $p(-3) = 0$

$\Rightarrow (k-1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow \quad 9(k-1) - 3k + 1 = 0$

$\Rightarrow \quad 9k - 9 - 3k + 1 = 0$

$\Rightarrow \quad 6k - 8 = 0$

$\therefore \quad k = \frac{4}{3}$

4. Given, 2 is the zero of the quadratic polynomial.

We have, $f(x) = x^2 + 3x + k$

$f(2) = 0$

$\Rightarrow (2)^2 + 3 \times 2 + k = 0$

$\Rightarrow \quad 4 + 6 + k = 0 \Rightarrow k = -10$

5. Let $f(x) = 3x^2 - 2kx + 2m$

2 and 3 are zeroes of $f(x)$

$f(2) = 0$ and $f(3) = 0$

$f(2) = 3(2)^2 - 2k \times 2 + 2m$

$0 = 12 - 4k + 2m$

$m - 2k = -6 \quad \ldots(i)$

$f(3) = 3(3)^2 - 2k \times 3 + 2m$

$0 = 27 - 6k + 2m$

$2m - 6k = -27 \quad \ldots(ii)$

From Eqs. (i) and (ii) we get, $m = 9$ and $k = \frac{15}{2}$

6. -4 is a zero of $x^2 - 2x - (7p + 3)$

Let $f(x) = x^2 - 2x - (7p + 3)$

$f(-4) = 0$

$\Rightarrow \quad 0 = (-4)^2 - 2 \times (-4) - (7p + 3)$

$\Rightarrow \quad 0 = 16 + 8 - (7p + 3)$

$\Rightarrow \quad 7p + 3 = 24$

$\Rightarrow \quad 7p = 24 - 3 = 21$

$\therefore \quad p = \frac{21}{7} = 3$

7. A linear polynomial intersects the X-axis at only one point.

8. For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like $\cup$ or open downwards like $\cap$ depending on whether $a > 0$ or $a < 0$. These curves are called parabolas.

So, option (d) cannot be possible.

Also, the curve of a quadratic polynomial crosses the X-axis on at most two points but in option (d) the curve crosses the X-axis on the three points, so it does not represent the quadratic polynomial

9. parabola

The graph of a quadratic function is always a parabola, irrespective of the values of x and y

10. Let α, β are the roots of the given equation

$2 - \sqrt{3}$

Comparing the given polynomial $x^2 - 4x + 1$ with the standard equation $ax^2 + bx + c$ we get $a = 1$, $b = -4$ and $c = 1$.

Sum of zeroes

$$\alpha + \beta = \frac{-b}{a} = 4$$

$$2 + \sqrt{3} + \beta = 4 \Rightarrow \beta = 2 - \sqrt{3}$$

11. Given,

Quadratic polynomial $(x^2 + 5x + 6)$

We have to find the zeroes of $x^2 + 5x + 6$

By factorization,

$$x^2 + 3x + 2x + 6 = 0$$
$$x(x+3) + 2(x+3) = 0$$
$$(x+3)(x+2) = 0$$
$$x = -3, x = -2$$

Hence, zeroes $x = -3$, $x = -2$

12. $p(z) = 0$

$z^2 - 27 = 0$

$z^2 = 27$

$(z)^2 = 27$

$z = \pm 3\sqrt{3}$

13. $abx^2 + (b^2 - ac)x - bc = 0$

$\Rightarrow abx^2 - acx + b^2x - bc = 0$

$\Rightarrow ax(bx - c) + b(bx - c) = 0$

$\Rightarrow \quad (ax + b)(bx - c) = 0$

Hence, $x = -\frac{b}{a}, \frac{c}{b}$ are zeroes.

14. The equation of a quadratic polynomial is x^2 – (sum of zeroes) x + product of zeroes

Sum of zeroes $= -2 + 5 = 3$

and product of zeroes $= -2 \times 5 = -10$

$\therefore$ Equation $= x^2 - 3x - 10.$

We know that, the zeroes do not change if the polynomial is divided or multiplied by a constant.

$\therefore kx^2 - 3kx - 10k$ will also have -2 and 5 as their zeroes. As k can take any real value there can be many polynomial having -2 and 5 as their zeroes

15. (True) Put 1 in $x^2 - 3x + 2$

$1 - 3 + 2 = 0$

1 satisfy the equation

Put 2 in $x^2 - 3x + 2$

$4 - 3 \times 2 + 2 = 0$

16. (True) Every real number is a zeroes of the zero polynomial hence the answer is true.

17. $p(x) = 0$

$x - 1 = 0$

$x = 1$

$g(x) = x^2 - 2x + 1$

According to question,

$p(x)$ is a factor of $g(x)$

$g(1) = 0$

$g(1) = 1 - 2 + 1 = 0$

So, $p(x)$ is a factor of $g(x)$

18. Given, 3 is a zero of polynomial $2x^2 + x + k$

Let $f(x) = 2x^2 + x + k$

$f(3) = 0$

$0 = 2(3)^2 + 3 + k$

$k = -(18 + 3)$

$k = -21$

19. α and β are zeroes of $x^2 - 7x + k$

We know that,

Sum of zeroes $(\alpha + \beta) = \frac{-b}{a} = \frac{7}{1}$ ...(i)

$\alpha \cdot \beta = \frac{c}{a} = k$

$\alpha - \beta = 1$ [Given] ...(ii)

From Eqs. (i) and (ii) we get $\alpha = 4$ and $\beta = 3$

$k = \alpha \cdot \beta$

$k = 4 \times 3 = 12$

20. (True) We know that,

Sum of zeroes of quadratic polynomial is

$$\frac{-b}{a} = -\left[\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}\right]$$

21. False

$f(\alpha) = f(\beta) = 0$

[$\because \alpha$ and β are the zeroes of $x^2 - 6x + a$]

$\alpha^2 - 6\alpha + a = 0$

$\beta^2 - 6\beta + a = 0$

$\therefore \quad \alpha + \beta = 6$ ...(i)

$3\alpha + 2\beta = 20$ [Given] ...(ii)

From Eqs. (i) and (ii) we get $\alpha = 8, \beta = -2$

$\beta^2 - 6\beta + a = 0$

$a = 6 \times (-2) - (-2)^2$

$= -12 - 4 = -16$

22. Let $p(x) = x^2 + ax + b.$

Now, product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Let α and β be the zeroes of $p(x)$.

$\therefore$ Product of zeroes $(\alpha \cdot \beta) = \frac{b}{1}$

$\Rightarrow \quad \alpha\beta = b$ ...(i)

Given that, one of the zeroes of a quadratic polynomial $p(x)$ is negative of the other.

$\therefore \quad \alpha\beta < 0$

So, $\quad b < 0$ [from Eq. (i)]

Hence, b should be negative

Put $a = 0$, then, $p(x) = x^2 + b = 0$

$$\Rightarrow \quad x^2 = -b$$

$$\Rightarrow \quad x = \pm\sqrt{-b} \quad [\because b < 0]$$

Hence, if one of the zeroes of quadratic polynomial $p(x)$ is the negative of the other, then it has no linear term i.e. $a = 0$ and the constant term is negative i.e. $b < 0$.

23. Given, p and q are the zeroes of quadratic equations

$$\therefore\ p + q = \frac{-2}{3} \text{ and } pq = -3$$

$$\therefore\ p - q = \pm\sqrt{(p+q)^2 - 4pq}$$

$$= \pm\sqrt{\left(\frac{-2}{3}\right)^2 - 4\times(-3)} = \pm\sqrt{\frac{4}{9} + 12} = \pm\frac{4\sqrt{7}}{3}$$

24. Sum of roots $= (\sqrt{2}+1) + \sqrt{2} - 1 = 2\sqrt{2}$

Product of roots $= (\sqrt{2}+1)(\sqrt{2}-1) = 2 - 1 = 1$

$\therefore$ Polynomial is

$x^2 -$ (sum of roots) $x +$ product of roots

$x^2 - 2\sqrt{2}x + 1$

25. α and β are the zeroes of $2y^2 + 7y + 5$

We know that,

$$\alpha + \beta = -\frac{b}{a} = -\frac{7}{2}$$

$$\alpha \cdot \beta = \frac{c}{a} = \frac{5}{2}$$

$$\Rightarrow (\alpha + \beta) + (\alpha \cdot \beta) = -\frac{7}{2} + \frac{5}{2} = \frac{-7+5}{2} = -1$$

26. α and β are the zeroes of quadratic polynomial.

$$f(x) = 3x^2 - 5x - 2$$

$\therefore$ We know that,

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-5)}{3} = \frac{5}{3}$$

$$\alpha \cdot \beta = \frac{c}{a} = -\frac{2}{3}$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\cdot\beta + \beta^2)$$

$$= (\alpha + \beta)\,[(\alpha + \beta)^2 - 3\alpha\cdot\beta]$$

$$= \frac{5}{3}\left[\left(\frac{5}{3}\right)^2 - 3\times\left(-\frac{2}{3}\right)\right]$$

$$= \frac{125}{27} + 2\times\frac{5}{3}$$

$$= \frac{125+90}{27} = \frac{215}{27}$$

27. α and β are the zeroes of $4x^2 + 3x + 7$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{3}{4}$$

$$\alpha\cdot\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{7}{4}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\cdot\beta} = \frac{-\frac{3}{4}}{\frac{7}{4}} = -\frac{3}{7}$$

28. Let $p(x) = x^2 + (a+1)x + b$

Given that, 2 and -3 are the zeroes of the quadratic polynomial $p(x)$

$\therefore \quad p(2) = 0$ and $p(-3) = 0$

$\Rightarrow \quad 2^2 + (a+1)(2) + b = 0$

$\Rightarrow \quad 4 + 2a + 2 + b = 0$

$\Rightarrow \quad 2a + b = -6 \quad$ ...(i)

and $(-3)^2 + (a+1)(-3) + b = 0$

$\Rightarrow \quad 9 - 3a - 3 + b = 0$

$\Rightarrow \quad 3a - b = 6 \quad$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$5a = 0 \Rightarrow a = 0$$

Put the value of a in Eq. (i), we get

$$2\times 0 + b = -6$$

$\Rightarrow \quad b = -6$

So, the required values are $a = 0$ and $b = -6$

29. We have, $\alpha + \beta = -3$ and $\alpha - \beta = -10$ (assuming $\alpha < \beta$)

$$\therefore \quad \alpha = -\frac{13}{2} \text{ and } \beta = \frac{7}{2}$$

Now, $\quad \alpha^2 - \beta^2 = 30$

30. Let the zeroes of the quadratic polynomial are α and β

$$\alpha + \beta = 8 \quad \text{...(i)}$$

and $\quad \alpha\cdot\beta = 12 \quad$ ...(ii)

Substitute value of $\beta = 8 - \alpha$ in Eq. (ii)

$$\alpha(8-\alpha) = 12$$

$$\alpha^2 - 8\alpha + 12 = 0$$

$$\alpha^2 - 6\alpha - 2\alpha + 12 = 0$$

$$\alpha(\alpha - 6) - 2(\alpha - 6) = 0$$

$$\alpha = 6, \alpha = 2$$

If $\alpha = 6$, then $\beta = 2$

If $\alpha = 2$, then $\beta = 6$

Hence,the two zeroes are 2 and 6.

31. m and n zeroes of $3x^2 + 11x - 4$,

$m + n = -\frac{11}{3}, m \cdot n = -\frac{4}{3}$

$\Rightarrow \quad \frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{m \cdot n} = \frac{(m+n)^2 - 2m \cdot n}{mn}$

$\Rightarrow \quad \frac{\left(\frac{-11}{3}\right)^2 - 2 \times \left(-\frac{4}{3}\right)}{-\frac{4}{3}} = -\frac{145}{12}$

32. Let the zeroes of the polynomial $3x^2 - 8x + 2k + 1$ be α, β

$\Rightarrow \quad \alpha + \beta = \frac{8}{3}$

We have, $\alpha = 7\beta$

$\Rightarrow \quad 7\beta + \beta = \frac{8}{3}$

$\Rightarrow \quad \beta = \frac{1}{3}$

$\Rightarrow \quad \alpha = \frac{7}{3}$ and $\beta = \frac{1}{3}$

$\alpha \cdot \beta = \frac{2k+1}{3}$

$\frac{7}{3} \times \frac{1}{3} = \frac{2k+1}{3}$

$2k = \frac{4}{3} \Rightarrow k = \frac{2}{3}$

33. Let α and β be the zeroes of $f(x)$.

Given, $\alpha^2 + \beta^2 = 20$

$(\alpha + \beta)^2 - 2\alpha\beta = 20 \quad \ldots(i)$

For the equation, $f(x) = x^2 - 4x + k$

$\alpha + \beta = 4$

$\alpha\beta = k$

Put these values in Eq. (i)

$\Rightarrow \quad (4)^2 - 2k = 20$

$\Rightarrow \quad 16 - 2k = 20$

$\Rightarrow \quad 2k = -4$

$\Rightarrow \quad k = -2$

34. Given, α and β are the zeroes of polynomial $x^2 - p(x+1) + c$ which can be written as $x^2 - px + c - p$.

So, sum of zeroes, $\alpha + \beta = p \quad \ldots(i)$

$\left[\because \text{sum of zeroes} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}\right]$

and product of zeroes, $\alpha\beta = c - p \quad \ldots(ii)$

$\left[\because \text{product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}\right]$

Also, $(\alpha + 1)(\beta + 1) = 0$ [given]

$\Rightarrow \quad \alpha\beta + (\alpha + \beta) + 1 = 0$

$\Rightarrow \quad c - p + p + 1 = 0$

[from Eqs. (i) and (ii)]

$\Rightarrow \quad c = -1$

35. Let α and β are the roots of given quadratic equation

$$x^2 - (k+6)x + 2(2k-1) = 0$$

Now, sum of roots $= \alpha + \beta$

$= -\left\{\frac{-(k+6)}{1}\right\} = k + 6$

Product of roots $= \alpha\beta = \frac{2(2k-1)}{1} = 2(2k-1)$

According to question,

Sum of roots (zeroes) $= \frac{1}{2}$

$\times$ products of roots (zeroes)

$\Rightarrow \quad k + 6 = \frac{1}{2} \times 2(2k-1)$

$\Rightarrow \quad k + 6 = 2k - 1$

$\Rightarrow \quad 6 + 1 = 2k - k \Rightarrow k = 7$

36. Let α and β are the zeroes of the polynomial $f(x) = 4x^2 - 27x + 3k^2$

$$\alpha + \beta = \frac{27}{4}, \alpha \cdot \beta = \frac{3k^2}{4}$$

As sum of zeroes would be equal to product of zeroes

$\alpha + \beta = \alpha\beta$

$\frac{27}{4} = \frac{3k^2}{4}$

$k^2 = 9$

$k = \pm 3$

37. α and β are the zeroes of $f(x) = x^2 - 4x + 3$

We know that,

$\alpha + \beta = -\frac{b}{a} = 4$

and $\quad \alpha \cdot \beta = \frac{c}{a} = 3$

$\Rightarrow \quad \alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta)$

$\Rightarrow \quad (\alpha\beta)^3(\alpha + \beta) = (3)^3(4)$

$= 4 \times 27 = 108$

38. Let α and β are the zeroes of quadratic polynomial

$\therefore$ Sum of zeroes, $\alpha+\beta=-5$ and product of zeroes, $\alpha\beta=6$ [given]

Now, required polynomial

$= x^2-(\text{sum of zeroes})\,x+\text{product of zeroes}$

$= x^2-(-5)x+6$

$= x^2+5x+6$

39. Sum of zeroes $=2\sqrt{7}+(-5\sqrt{7})=-3\sqrt{7}$

Product of zeroes $=2\sqrt{7}\times(-5\sqrt{7})=-70$

The quadratic polynomial are

$x^2-(\text{sum of zeroes})\,x+\text{product of zeroes}$

$x^2-(-3\sqrt{7})x-70$

$= x^2+3\sqrt{7}x-70$

40. Sum of zeroes $=3+\sqrt{2}+3-\sqrt{2}=6$

Product of zeroes $=(3+\sqrt{2})(3-\sqrt{2})$

$=9-2=7$

The quadratic polynomial is

$= x^2-(\text{sum of zeroes})\,x+\text{product of zeroes}$

$= x^2-6x+7$

41. Given quadratic polynomial

$$f(x)=x^2+x-2$$

$\alpha+\beta=-1$ and $\alpha\cdot\beta=-2$

Sum of zeroes i.e., $(2\alpha+1, 2\beta+1)$

$=2\alpha+1+2\beta+1$

$=2(\alpha+\beta+1)$

$=2(-1+1)=0$

Product of zeroes i.e., $(2\alpha+1, 2\beta+1)$

$=(2\alpha+1)(2\beta+1)$

$=4\alpha\beta+2(\alpha+\beta)+1$

$=4(-2)+2(-1)+1$

$=-8-2+1=-9$

The quadratic polynomial whose zeroes are $(2\alpha+1), (2\beta+1)$ are

$= x^2-(\text{sum of zeroes})\,x+\text{product of zeroes}$

$= x^2-0\times x+(-9)=x^2-9$

42. Let $p(x)=x^2-5$

For finding the zeroes of $p(x)$, put $p(x)=0$.

$\therefore\quad x^2-5=0 \Rightarrow x=\pm 5$

Let $\alpha=5$ and $\beta=-5$

Now, $1+\alpha=1+5=6$ and $1+\beta=1-5=-4$

Thus, 6 and -4 are the zeroes of new quadratic polynomial.

Therefore the new quadratic polynomial is

$x^2-(\text{sum of zeroes})x+\text{product of zeroes}$

$\therefore$ Required polynomial

$= x^2-[6+(-4)]x+6\times(-4)$

$= x^2-2x-24$

43. Let $f(x)=kx^2+x+k$

For equal roots. Its discriminant should be zero i.e. $D=b^2-4ac=0$

$\Rightarrow \quad 1-4k\cdot k=0 \Rightarrow k=\pm\frac{1}{2}$

So, for two values of k, given quadratic polynomial has equal zeroes.

44. $p(x)=5x^3-3x^2+7x+2$

(A) $p(1)=5-3+7+2=11$

(B) $p(0)=5(0)^3-3(0)^2+7\times 0+2=2$

(C) $p(5)=5(5)^3-3(5)^2+7\times 5+2$

$=625-75+35+2=587$

(D) $p(-1)=5(-1)^3-3(-1)^2+7\times(-1)+2$

$=-5-3-7+2=-13$

(E) $p(-2)=5(-2)^3-3(-2)^2+7\times(-2)+2$

$=-40-12-14+2=-64$

45. $\alpha+\beta=-\left(-\frac{4}{2}\right)=2$ and $\alpha\cdot\beta=\frac{5}{2}$

(A) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\cdot\beta}=\frac{2\times 2}{5}=\frac{4}{5}$

(B) $(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\cdot\beta$

$=(\alpha+\beta)^2-4\alpha\beta$

$=(2)^2-4\times\frac{5}{2}$

$=4-10=-6$

(C) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$

$=\frac{(\alpha+\beta)^2-2\alpha\cdot\beta}{(\alpha\cdot\beta)^2}$

$=\frac{(2)^2-2\times\frac{5}{2}}{\left(\frac{5}{2}\right)^2}$

$=\frac{4-5}{\frac{25}{4}}=-\frac{4}{25}$

(D) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\cdot\beta}$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{(2)^2-2\times\frac{5}{2}}{\frac{5}{2}}$$

$$=\frac{4-5}{\frac{5}{2}}=\frac{-2}{5}$$

46. **Assertion** Consider the given polynomial x^2+4x+5

Because the degree of the polynomial is 2. It is a quadratic polynomial

We know that, the quadratic polynomial has atmost two zeroes

$\therefore x^2+4x+5$ has two zeroes

$\therefore$ Assertion is true

Reason Clearly, Reason is true

Hence, both Assertion and Reason are true and Reason is a correct explanation of Assertion

47. The expression is calculated by splitting the middle term

Let be $f(x)=\frac{3}{2}x^2-8x-\frac{35}{2}$

$$=\frac{1}{2}[3x^2-16x-35]$$

$$=\frac{1}{2}(3x^2-21x+5x-35]$$

$$=\frac{1}{2}[3x(x-7)+5(x-7)]$$

$$=\frac{1}{2}[(3x+5)(x-7)]$$

Assertion is true but Reason is false.

48. Let $\alpha=4$ and $\beta=3$

Then $\alpha+\beta=7$ and $\alpha\cdot\beta=12$

$\therefore$ Required polynomial $=x^2$

$$-(\alpha+\beta)x+\alpha\cdot\beta=x^2-7x-12$$

So, that Assertion is true.

Both the Assertion and Reason are true and Reason is a correct explanation of Assertion.

49. **Assertion** Given, $f(x)=x^2-4x-5$

Splitting the middle term

$$f(x)=x^2-5x+x-5$$
$$=x(x-5)+1(x-5)$$
$$=(x+1)(x-5)$$
$$f(x)=0$$
$x=-1$ and $x=5$ (True)

Reason We know that, the polynomial is

$x^2-(\alpha+\beta)x+\alpha\cdot\beta$

$x^2-(2+\sqrt{3}+2-\sqrt{3})x+(2+\sqrt{3})\cdot(2-\sqrt{3})$

$x^2-4x+(4-3)$

x^2-4x+1

Assertion is true and Reason is false

50. Let $f(x)=x^2+2x-15$

$\alpha+\beta=-2$

and $\alpha\cdot\beta=-15$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\cdot\beta}$$
$$=\frac{-2}{-15}=\frac{2}{15}$$

Assertion is true.

Reason is true.

So, Assertion is true, Reason is true; Reason is a correct explanation of Assertion.

51. Let $\alpha,\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

$$\alpha\cdot\frac{1}{\alpha}=\frac{4k}{k+18}$$

$$\Leftrightarrow \quad \frac{4k}{k+18}=1$$

$$4k=k+18$$
$$3k=18$$
$$k=6$$

Assertion is true, Reason is true, and Reason is not correct explanation for assertion

52. **Assertion** Let α,β be the zeroes of given polynomial x^2+px+q

$\therefore \quad \alpha-\beta=1 \quad$ ...(i)

and $\quad \alpha+\beta=-p$

$\alpha\cdot\beta=q \quad$ ...(ii)

from Eq. (i), squaring $(\alpha-\beta)^2=1$

$$\Rightarrow \quad (\alpha+\beta)^2 - 4\alpha\beta = 1$$
$$\Rightarrow \quad (-p)^2 - 4q = 1 \quad \text{(using (ii))}$$
$$\Rightarrow \quad p^2 - 1 = 4q$$

Reason Given, $p(x) = (x-a)(x-b) - c$
$$= x^2 - (a+b)x + (ab-c)$$
$$\therefore \quad \alpha + \beta = -\frac{\{-(a+b)\}}{1} = a + b$$
$$\Rightarrow \quad a + b = \alpha + \beta \quad \ldots\text{(i)}$$
$\{\because \alpha$ and β are zeroes of $p(x)\}$

and $\alpha\beta = \frac{(ab-c)}{1} = ab - c$
$$\Rightarrow \quad ab = \alpha\beta + c$$

Now, family of polynomials having a and b as its zeroes is given by $k[x^2 - (a+b)x + ab]$
$$= k[x^2 - (\alpha+\beta)x + (\alpha\beta + c)\}$$
[Using Eqs. (i) and (ii)]

Now, Taking $k = 1$, the polynomial with a and b as its zeroes
$$= x^2 - (\alpha+\beta)x + \alpha\beta + c$$
$$= (x-\alpha)(x-\beta) + c$$

Assertion is the true, Reason is true but Reason is not a correct explanation for Assertion.

53. Let the roots are α and $-\alpha$

$$\text{Sum of zero} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$\alpha + (-\alpha) = \frac{-4k}{k^2+4}$$
$$0 = 4k$$

Thus, $k = 0$

Hence, Assertion is true but Reason is false.

54. Let $f(x) = 3x^2 + 11x - 4$
$$= 3x^2 + 12x - x - 4$$
$$= 3x(x+4) - 1(x+4)$$
$$= (3x-1)(x+4)$$

Let $m = \frac{1}{3}$

and $n = -4$

$12(m^2 + n^2) + 145mn$
$$= 12\left(\frac{1}{9} + 16\right) + 145 \times \left(\frac{1}{3}\right) \times (-4)$$
$$= 12\left(\frac{1+144}{9}\right) - \frac{145 \times 4}{3}$$
$$= \frac{12 \times 145}{9} - \frac{145 \times 4}{3} = 0$$

Given, α and β are roots of $ax^2 + by + c$
$$\therefore \alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$$
$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$
$$= \frac{(\alpha+\beta)^2}{\alpha\beta} - 2 = \frac{b^2}{ac} - 2$$

Hence, Assertion is true but Reason is false.

55. Let $g(x) = x^2 + x - 12$
$$= x^2 + 4x - 3x - 12$$
$$= x(x+4) - 3(x+4)$$
$$= (x-3)(x+4)$$
$$p(3) = 0 = p(-4)$$
$$p(3) = 3^3 + a(3)^2 + b(3) - 84 = 0$$
$$27 + 9a + 3b - 84 = 0$$
$$9a + 3b = 57 \quad \ldots\text{(i)}$$
$$p(-4) = (-4)^3 + a(-4)^2 + b(-4) - 84 = 0$$
$$\Rightarrow \quad -64 + 16a - 4b - 84 = 0$$
$$4a - b = 37$$
$$12a - 3b = 111 \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii)
$$21a = 168$$
$$a = 8$$

Substitute $a = 8$ in Eq. (i)
$$9 \times 8 + 3 \times b = 57$$
$$72 + 3b = 57$$
$$3b = -15$$
$$b = -5$$

Hence, Assertion is false but Reason is true.

56. (i) Graph of a quadratic polynomial is parabolic in shape.

(ii) $(x^2 - 64)$ is expression of the polynomial represented because $A(-8, 0)$ and $B(8, 0)$ i.e. on this quadratic equation $(x^2 - 64)$.

(iii) Let $f(x) = x^2 - 64$
$$f(8) = (8)^2 - 64 = 64 - 64 = 0$$

(iv) Given, $x^2 - 2x - 4$

$$\text{Sum of zeroes} = -b/a = \frac{-(-2)}{1} = 2$$

(v) Given, $ax^2 + 5x - 3a$

According to question,
Sum of zero = Product of zero

$$\frac{-5}{a} = \frac{-3a}{a}$$

$$\therefore \quad a = \frac{5}{3}$$

57. (i) In the standard form of quadratic polynomial $ax^2 + bx + c$; 'a' is a non-zero real number, and b and c are any real number.

(ii) In a quadratic polynomial, if roots are equal, then discriminant, $D = 0$.

(iii) Given, α and $\frac{1}{\alpha}$ are the zeroes of quadratic polynomial $2x^2 - x + 8k$.

Now, product of zeroes,

$$\alpha \times \frac{1}{\alpha} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Rightarrow \quad 1 = \frac{8k}{2}$$

$$\Rightarrow \quad k = \frac{2}{8} = \frac{1}{4}$$

(iv) Given equation is $x^2 + 1 = 0$.

$$x^2 = -1$$

It gives imaginary roots.

Hence, it is neither touches nor intersects X-axis.

(v) Given, sum of roots $= -p$

and product of roots $= -\frac{1}{p}$

$\therefore$ Quadratic polynomial

$$= k\,[x^2 - (\text{Sum of roots})\,x + \text{Product of roots}]$$

$$= k\left[x^2 - (-p)x + \left(-\frac{1}{p}\right)\right]$$

$$= k\left(x^2 + px - \frac{1}{p}\right)$$

58. (i) Given, $x^2 - 23x + 120 = p(x)$

Let α, β are the zeroes of $p(x)$

$$x^2 - 23x + 120 = 0$$

$$\Rightarrow x^2 - 15x - 8x + 120 = 0$$

$$\Rightarrow x(x - 15) - 8(x - 15) = 0$$

$$\Rightarrow \quad (x - 8)(x - 15) = 0$$

$$\Rightarrow \quad x = 8,\ x = 15$$

(ii) $\alpha + \beta + \alpha \cdot \beta$

(Sum of zeroes) + (Product of zeroes)

$$= -(-23) + 120$$

$$= 23 + 120 = 143$$

(iii) $p(x) = x^2 - 23x + 120$

$$p(2) = (2)^2 - 23 \times 2 + 120$$

$$p(2) = 4 - 46 + 120 = 78$$

(iv) Given, $x^2 + 3x - 10$

$$\alpha + \beta = -3,\ \alpha \cdot \beta = -10$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \cdot \beta}$$

$$= \frac{-3}{-10} = \frac{3}{10}$$

(v) Given, $q(x) = kx^2 + 2x + 3k$

Sum of zeroes = equal to their product

$$\frac{-2}{k} = \frac{3k}{k}$$

$$k = -2/3$$

59. (i) The shape of given poses are parabolic.

(ii) The graph of parabola opens downwards, if $a < 0$.

(iii) In the given graph, we see that curve cut the X-axis at exactly one point.

Hence, number of zeroes in the given polynomial is 1.

(iv) The curve intersect X-axis at points $x = -2$ and $x = 4$.

Hence, two zeroes in the given graph are -2 and 4.

(v) Let $p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$$= 4\sqrt{3}x^2 + (8 - 3)x - 2\sqrt{3}$$

[by splitting middle term]

$$= 4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3}$$

$$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$$

$$= (4x - \sqrt{3})(\sqrt{3}x + 2)$$

For finding the zeroes, put $p(x) = 0$

$\therefore \quad (4x - \sqrt{3})(\sqrt{3}x + 2) = 0$

$\Rightarrow \quad 4x - \sqrt{3} = 0$ and $\sqrt{3}x + 2 = 0$

$\Rightarrow \quad x = \frac{\sqrt{3}}{4}$ and $x = -\frac{2}{\sqrt{3}}$

60. (i) Point $A(-7, 0)$ and $B(10, 0)$ lies on this parabolic shape graph.

Its mean -7 and 10 are zeroes of the polynomial $(x + 7)(x - 10)$

(ii) $(x + 7)(x - 10)$

$\Rightarrow \quad x^2 + 7x - 10x - 70 \Rightarrow x^2 - 3x - 70$

(iii) Product of zeroes $= c / a$

Given, $(x^2 - 3x - 70)$

Product of zeroes $= -70$

(iv) Let be, $f(x) = 9x^2 - 5$

$f(x) = 0$

$9x^2 - 5 = 0$

$x^2 = 5/9$

$x = \pm\sqrt{5/9} = \pm\sqrt{5}/3$

(v) Given, $f(x) = x^2 - 13x + 1$

$f(3) = (3)^2 - 13 \times 3 + 1$

$f(3) = 9 + 1 - 39 = -29$

61. (i) The shape of the path traced shown in the given figure is the form of parabola.

(ii) The graph of parabola opens downwards, if $a < 0$.

(iii) In the given graph, we see that curve intersect the X-axis at three points. Hence, number of zeroes in the given polynomial are 3.

(iv) The given curve intersect the X-axis at points $x = -3, -1$ and 2.

Hence, three zeroes in thze given graph are $-3, -1, 2$

(v) Since, given polynomial has three zeroes.

So, it will be a cubic polynomial.

Now, sum of zeroes $= -3 - 1 + 2 = -2$

Sum of product of two zeroes at a time

$= -3 \times (-1) + (-1) \times 2 + 2 \times (-3)$

$= 3 - 2 - 6$

$= -5$

and product of all zeroes $= -3 \times -1 \times 2 = 6$

$\therefore$ Required cubic polynomial

$= x^3 -$ (Sum of zeroes) x^2

$+$ (Sum of product of two zeroes at a time) $x -$ (Product of three zeroes)

$= x^3 - (-2)x^2 + (-5)x - (6)$

$= x^3 + 2x^2 - 5x - 6$